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Completely Entangled State and Simultaneous Eigenstate in a Finite Dimensional Space

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Abstract It is shown that a completely entangled state belonging to a finite dimensional Hilbert space is equivalent to a simultaneous eigenstate of two unitary operators. These operators are exponentials of sum and difference of two Hermitian operators of a two-mode system that are complementary with each other.

Keywords Entangled state · Simultaneous eigenstate · Complementarity

1 Introduction

The completely entangled state $|\Psi(x,p)\rangle$ of continuous variables, which is called the EPR state [1], is a simultaneous eigenstates of position-difference operator $\hat{x}_a - \hat{x}_b$ and momentum-sum operator $\hat{p}_a + \hat{p}_b$ of a two-mode system such that $(\hat{x}_a - \hat{x}_b)|\Psi(x,p)\rangle = x|\Psi(x,p)\rangle$ and $(\hat{p}_a + \hat{p}_b)|\Psi(x,p)\rangle = p|\Psi(x,p)\rangle$, where $\hat{x}_a$ ($\hat{x}_b$) and $\hat{p}_a$ ($\hat{p}_b$) are position and momentum operators of the system a (b). The continuous variable EPR state is explicitly written in the following form:

$$|\Psi(x,p)\rangle = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dy\, |x+y\rangle \otimes |y\rangle e^{ipy}, \tag{1}$$

where $|x\rangle$ is an eigenstate of the position operator. The set of the EPR states $|\Psi(x,p)\rangle$ becomes a complete orthonormal basis of the two-mode system. Furthermore the set of projection operators $\hat{\Pi}(x,p) = |\Psi(x,p)\rangle\langle\Psi(x,p)|$ describes the simultaneous measurement of position and momentum of the two-mode system. The EPR state $|\Psi(x,p)\rangle$ plays an important role in the continuous variable quantum information processing such as the quantum teleportation and the quantum dense coding [2].

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For a two-qubit system, the Bell states that are completely entangled become a complete orthonormal basis in four dimensional Hilbert space, where the Bell states are given by

$$|\Psi_{00}\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}, \qquad |\Psi_{01}\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}, \tag{2}$$

$$|\Psi_{10}\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}}, \qquad |\Psi_{11}\rangle \frac{|01\rangle - |10\rangle}{\sqrt{2}}. \tag{3}$$

When $|0\rangle$ and $|1\rangle$ are eigenstates of the Pauli matrix $\hat{\sigma}^z$ such that $\hat{\sigma}^z|0\rangle = |0\rangle$ and $\hat{\sigma}^z|1\rangle = -|1\rangle$, the Bell states are simultaneous eigenstate of Hermitian/unitary operators $\hat{\sigma}^x \otimes \hat{\sigma}^x$ and $\hat{\sigma}^z \otimes \hat{\sigma}^z$ such that $(\hat{\sigma}^x \otimes \hat{\sigma}^x)|\Psi_{jk}\rangle = (-1)^j|\Psi_{jk}\rangle$ and $(\hat{\sigma}^z \otimes \hat{\sigma}^z)|\Psi_{jk}\rangle = (-1)^k|\Psi_{jk}\rangle$. This fact shows that the Bell measurement is equivalent to observing whether two $1/2$-spins are parallel or anti-parallel in x-direction and in z-direction. The Bell state also plays an essential role in the discrete variable quantum information processing [3].

These examples imply that there exists some correspondence between a completely entangled state and simultaneous eigenstate. In this paper, we investigate such correspondence and we show that a completely entangled state belonging to a finite dimensional Hilbert space is equivalent to a simultaneous eigenstates of two unitary operators. Furthermore we find a quantum measurement that is described by separable positive operator-valued measure, but that cannot not be performed by a local operation with classical communication. Although entanglement of quantum states has bee studied extensively [4], this paper will provide a new insight into entanglement.

2 Complementary Observables in a Finite Dimensional Space

We first introduce complementary operators defined on a finite dimensional Hilbert space. For this purpose, we suppose that $\{|a_0\rangle, |a_1\rangle, \dots, |a_{N-1}\rangle\}$ is a complete orthonormal system of an N dimensional Hilbert space $\mathcal{H}_N$. Then, performing a discrete Fourier transformation, we obtain another complete orthonormal system $\{|b_0\rangle, |b_1\rangle, \dots, |b_{N-1}\rangle\}$, where $|a_j\rangle$ and $|b_k\rangle$ are connected by the relation,

$$|b_j\rangle = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \omega^{jk} |a_k\rangle, \tag{4}$$

$$|a_j\rangle = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \omega^{*jk} |b_k\rangle. \tag{5}$$

In this equation, $\omega = e^{2\pi i/N}$ is the primitive Nth root of unity. It is easy to verify the relation,

$$\frac{1}{N} \sum_{m=0}^{N-1} \omega^{(j-k)m} = \delta_{j,k \bmod N}, \tag{6}$$

where $\delta_{j,k \bmod N} = 1$ for $j = k \pmod N$ and otherwise zero.

Using the sets $\{|a_0\rangle, |a_1\rangle, \dots, |a_{N-1}\rangle\}$ and $\{|b_0\rangle, |b_1\rangle, \dots, |b_{N-1}\rangle\}$, we introduce two Hermitian operators $\hat{A}$ and $\hat{B}$ by $\hat{A} = \sum_{k=0}^{N-1} k|a_k\rangle\langle a_k|$ and $\hat{B} = \sum_{k=0}^{N-1} k|b_k\rangle\langle b_k|$ which are complementary with each other. These operators satisfy the commutation relation [5, 6],

$$[\hat{A},\hat{B}] = \sum_{\substack{j=0 \\ (j\neq k)}}^{N-1}\sum_{k=0}^{N-1} \frac{(j-k)|a_j\rangle\langle a_k|}{\omega^{j-k}-1} = \sum_{\substack{j=0 \\ (j\neq k)}}^{N-1}\sum_{k=0}^{N-1} \frac{(j-k)|b_k\rangle\langle b_j|}{\omega^{j-k}-1}. \tag{7}$$

Furthermore we define unitary exponential operators $e^{(2\pi i/N)j\hat{A}}$ and $e^{(2\pi i/N)j\hat{B}}$. These operators satisfy the relations,

$$e^{\pm(2\pi i/N)j\hat{A}}|a_k\rangle = \omega^{\pm jk}|a_k\rangle, \tag{8}$$

$$e^{\pm(2\pi i/N)j\hat{A}}|b_k\rangle = |b_{k\pm j \bmod N}\rangle, \tag{9}$$

$$e^{\pm(2\pi i/N)j\hat{B}}|b_k\rangle = \omega^{\pm jk}|b_k\rangle, \tag{10}$$

$$e^{\pm(2\pi i/N)j\hat{B}}|a_k\rangle = |b_{k\mp j \bmod N}\rangle. \tag{11}$$

In particular, we can obtain the expressions,

$$e^{-(2\pi i/N)\hat{A}} = \sum_{k=0}^{N-1} |b_{k-1 \bmod N}\rangle\langle b_k|, \tag{12}$$

$$e^{(2\pi i/N)\hat{B}} = \sum_{k=0}^{N-1} |a_{k-1 \bmod N}\rangle\langle a_k|. \tag{13}$$

The unitary operators $e^{(2\pi i/N)j\hat{A}}$ and $e^{(2\pi i/N)j\hat{B}}$ satisfy the Weyl commutation relation [5, 6],

$$e^{(2\pi i/N)j\hat{B}}e^{(2\pi i/N)k\hat{A}} = \omega^{jk}e^{(2\pi i/N)k\hat{A}}e^{(2\pi i/N)j\hat{B}}. \tag{14}$$

The Hermitian operators $\hat{A}$ and $\hat{B}$ include the Pegg-Barnett phase operator and its canonically conjugate operator defined on the finite dimensional Hilbert space [7, 8].

3 Completely Entangled State and Simultaneous Eigenstate

We next introduce a completely entangled state $|\Psi\rangle$ which is an element of an $N\times N$ dimensional Hilbert space $\mathcal{H}_N\otimes\mathcal{H}_N$,

$$|\Psi\rangle = \frac{1}{\sqrt{N}}\sum_{k=0}^{N-1}|a_k\rangle\otimes|a_k\rangle. \tag{15}$$

Furthermore we define a unitary operator $\hat{D}_{jk}$ in terms of the Hermitian operators $\hat{A}$ and $\hat{B}$,

$$\hat{D}_{jk} = e^{-(2\pi i/N)j\hat{B}}e^{(2\pi i/N)k\hat{A}}, \tag{16}$$

which satisfies the relations [9],

$$\frac{1}{N}\mathrm{Tr}(\hat{D}_{jk}\hat{D}_{lm}) = \delta_{j,l \bmod N}\delta_{k,m \bmod N}, \tag{17}$$

$$\frac{1}{N}\sum_{j=0}^{N-1}\sum_{k=0}^{N-1}\hat{D}_{jk}\hat{X}\hat{D}_{jk}^{\dagger} = (\mathrm{Tr}\hat{X})\hat{1}, \tag{18}$$

for any operator $\hat{X}$ defined on the Hilbert space $\mathcal{H}_N$. The unitary operator $\hat{D}_{jk}$ is equivalent to the generalized Pauli matrix [10, 11]. Using the unitary operator $\hat{D}_{jk}$, we can construct a complete orthonormal basis $\{|\Psi_{jk}\rangle \mid j,k = 0,1,\ldots,N-1\}$ of completely entangled states in the $N \times N$ dimensional Hilbert space $\mathcal{H}_N \otimes \mathcal{H}_N$, where $|\Psi_{jk}\rangle$ is defined by

$$\begin{aligned} |\Psi_{jk}\rangle &= (\hat{D}_{jk}\otimes\hat{1})|\Psi\rangle \\ &= \frac{1}{\sqrt{N}}\sum_{m=0}^{N-1}|a_{m+j \bmod N}\rangle\otimes|a_m\rangle e^{(2\pi i/N)km}, \end{aligned} \tag{19}$$

where the orthogonality relation $\langle\Psi_{jk}|\Psi_{lm}\rangle = \delta_{j,l}\delta_{k,m}$ and the completeness relation $\sum_{j=0}^{N-1}\sum_{k=0}^{N-1}|\Psi_{jk}\rangle\langle\Psi_{jk}| = \hat{1}\otimes\hat{1}$ are satisfied. We can also express the completely entangled state $|\Psi_{jk}\rangle$ in terms of $|b_k\rangle$'s as

$$|\Psi_{jk}\rangle = \frac{1}{\sqrt{N}}\sum_{m=0}^{N-1}|b_m\rangle\otimes|b_{k-m \bmod N}\rangle e^{-(2\pi i/N)jm}. \tag{20}$$

Using the Hermitian operators $\hat{A}$ and $\hat{B}$, we introduce four Hermitian operators $\hat{A}_k$ and $\hat{B}_k$ $(k = 1,2)$ defined on the Hilbert space $\mathcal{H}_N \otimes \mathcal{H}_N$ by the relations,

$$\hat{A}_1 = \hat{A}\otimes\hat{1}, \qquad \hat{A}_2 = \hat{1}\otimes\hat{A}, \qquad \hat{B}_1 = \hat{B}\otimes\hat{1}, \qquad \hat{B}_2 = \hat{1}\otimes\hat{B}, \tag{21}$$

in terms of which we define unitary exponential operators of difference-operator $\hat{A}_1 - \hat{A}_2$ and sum-operator $\hat{B}_1 + \hat{B}_2$ by $e^{(2\pi i/N)m(\hat{A}_1-\hat{A}_2)}$ and $e^{(2\pi i/N)n(\hat{B}_1+\hat{B}_2)}$ for any integers m and n. These unitary operators commute with each other,

$$[e^{(2\pi i/N)m(\hat{A}_1-\hat{A}_2)}, e^{(2\pi i/N)n(\hat{B}_1+\hat{B}_2)}] = 0. \tag{22}$$

The completely entangled state $|\Psi_{jk}\rangle$ given by (19) is a simultaneous eigenstate of $e^{(2\pi i/N)m(\hat{A}_1-\hat{A}_2)}$ and $e^{(2\pi i/N)n(\hat{B}_1+\hat{B}_2)}$ such that

$$e^{(2\pi i/N)m(\hat{A}_1-\hat{A}_2)}|\Psi_{jk}\rangle = \omega^{jm}|\Psi_{jk}\rangle, \tag{23}$$

$$e^{(2\pi i/N)n(\hat{B}_1+\hat{B}_2)}|\Psi_{jk}\rangle = \omega^{kn}|\Psi_{jk}\rangle. \tag{24}$$

The projection operators $\hat{W}_j^A$ and $\hat{W}_k^B$ into the eigenspaces of $e^{(2\pi i/N)m(\hat{A}_1-\hat{A}_2)}$ and $e^{(2\pi i/N)n(\hat{B}_1+\hat{B}_2)}$ are given by

$$\hat{W}_j^A = \sum_{k=0}^{N-1}|\Psi_{jk}\rangle\langle\Psi_{jk}|$$

$$= \sum_{m=0}^{N-1} |a_{j+m \bmod N}\rangle\langle a_{j+m \bmod N}| \otimes |a_m\rangle\langle a_m|, \tag{25}$$

$$\hat{W}_k^B = \sum_{j=0}^{N-1} |\Psi_{jk}\rangle\langle\Psi_{jk}|$$

$$= \sum_{n=0}^{N-1} |b_n\rangle\langle b_n| \otimes |b_{k-n \bmod N}\rangle\langle b_{k-n \bmod N}|. \tag{26}$$

Since these projectors commute with each other, $[\hat{W}_j^A, \hat{W}_k^B] = 0$, and are normalized as $\sum_{j=0}^{N-1} \hat{W}_j^A = \sum_{k=0}^{N-1} \hat{W}_k^B = \hat{1} \otimes \hat{1}$, the sets $\{\hat{W}_j^A \,|\, j = 0, 1, \ldots, N-1\}$ and $\{\hat{W}_k^B \,|\, k = 0, 1, \ldots, N-1\}$ describe simultaneous measurement of some commutable observables. Although the projection operators $\hat{W}_j^A$ and $\hat{W}_k^B$ are separable forms, the measurement cannot be performed by means of local operation and classical communication (LOCC) since these projectors satisfy the relation $\hat{W}_j^A \hat{W}_k^B = \hat{W}_k^B \hat{W}_j^A = |\Psi_{jk}\rangle\langle\Psi_{jk}|$. If the measurement were possible by LOCC, the completely entangled state $|\Psi_{jk}\rangle$ could have been created from any quantum state defined on the Hilbert space $\mathcal{H}_N \otimes \mathcal{H}_N$ by LOCC.

Using an appropriate basis, e.g. $\{|a_0\rangle, |a_1\rangle, \ldots, |a_{N-1}\rangle\}$, we can express any completely entangled state $|\Psi\rangle$ belonging to the Hilbert space $\mathcal{H}_N \otimes \mathcal{H}_N$ in the form of $|\Psi\rangle = (1/\sqrt{N}) \sum_{k=0}^{N-1} |a_k\rangle \otimes |a_k\rangle$. In this case, we introduce two Hermitian operators by $\hat{A} = \sum_k^{N-1} k|a_k\rangle\langle a_k|$ and $\hat{B} = \sum_{k=0}^{N-1} k|b_k\rangle\langle b_k|$ with $|b_k\rangle$ being the discrete Fourier transform of $|a_k\rangle$, and we construct four Hermitian operators $\hat{A}_k$ and $\hat{B}_k$ $(k = 1, 2)$ by (21). Then we can see that the completely entangled state $|\Psi\rangle$ is the simultaneous eigenstate of the two unitary operators $e^{(2\pi i/N)m(\hat{A}_1 - \hat{A}_2)}$ and $e^{(2\pi i/N)n(\hat{B}_1 + \hat{B}_2)}$ with eigenvalue of unity, that is, $e^{(2\pi i/N)m(\hat{A}_1 - \hat{A}_2)}|\Psi\rangle = |\Psi\rangle$ and $e^{(2\pi i/N)n(\hat{B}_1 + \hat{B}_2)}|\Psi\rangle = |\Psi\rangle$. This result shows that any completely entangled state is a simultaneous eigenstate of the two unitary operators. Furthermore, the set $\{|\Psi_{jk}\rangle = (\hat{D}_{jk} \otimes \hat{1})|\Psi\rangle \,|\, j, k = 0, 1, \ldots, N-1\}$ with the unitary operator $\hat{D}_{jk}$ given by (16) becomes a complete orthonormal basis of the $N \times N$ dimensional Hilbert space $\mathcal{H}_N \otimes \mathcal{H}_N$. It is easy to see that the completely entangled state $|\Psi_{jk}\rangle$ is a simultaneous eigenstate of $e^{(2\pi i/N)m(\hat{A}_1 - \hat{A}_2)}$ and $e^{(2\pi i/N)n(\hat{B}_1 + \hat{B}_2)}$ such that $e^{(2\pi i/N)m(\hat{A}_1 - \hat{A}_2)}|\Psi_{jk}\rangle = \omega^{jm}|\Psi_{jk}\rangle$ and $e^{(2\pi i/N)n(\hat{B}_1 + \hat{B}_2)}|\Psi_{jk}\rangle = \omega^{kn}|\Psi_{jk}\rangle$ with $\omega = e^{2\pi i/N}$.

We next show that the simultaneous eigenstate of the commutable unitary operators $e^{(2\pi i/N)m(\hat{A}_1 - \hat{A}_2)}$ and $e^{(2\pi i/N)n(\hat{B}_1 + \hat{B}_2)}$ is a completely entangled. It is obvious that $(e^{(2\pi i/N)m(\hat{A}_1 - \hat{A}_2)})^N = \hat{1} \otimes \hat{1}$ and $(e^{(2\pi i/N)n(\hat{B}_1 + \hat{B}_2)})^N = \hat{1} \otimes \hat{1}$. Hence the eigenvalues of these operators are Nth roots of unity. We denote as $|\Psi\rangle$ the simultaneous eigenstate with eigenvalue of unity, such that $e^{(2\pi i/N)m(\hat{A}_1 - \hat{A}_2)}|\Psi\rangle = |\Psi\rangle$ and $e^{(2\pi i/N)n(\hat{B}_1 + \hat{B}_2)}|\Psi\rangle = |\Psi\rangle$. Using the complete orthonormal basis $\{|a_0\rangle, |a_1\rangle, \ldots, |a_{N-1}\rangle\}$ of the N dimensional Hilbert space $\mathcal{H}_N$, we can express this eigenstate as $|\Psi\rangle = \sum_{j=0}^{N-1} \sum_{k=0}^{N-1} C_{jk}|a_j\rangle \otimes |a_k\rangle$. We can see that the eigenvalue equation $e^{(2\pi i/N)m(\hat{A}_1 - \hat{A}_2)}|\Psi\rangle = |\Psi\rangle$ yields the relation $C_{jk}\omega^{(j-k)m} = C_{jk}$ for any integer m. Hence we find that $C_{jk} = C_j \delta_{j,k}$ and thus we obtain $|\Psi\rangle = \sum_{k=0}^{N-1} C_k|a_k\rangle \otimes |a_k\rangle$. Furthermore the eigenvalue equation $e^{(2\pi i/N)n(\hat{B}_1 + \hat{B}_2)}|\Psi\rangle = |\Psi\rangle$ provides the equality $C_{j+m \bmod N} = C_j$ for any integer m, which yields the equality $C_0 = C_1 = \cdots = C_{N-1}$. Therefore, ignoring an unimportant phase factor and imposing the normalization condition, we can obtain the simultaneous eigenstate in the form of $|\Psi\rangle = (1/\sqrt{N}) \sum_{k=0}^{N-1} |a_k\rangle \otimes |a_k\rangle$ which is a completely entangled state. The other eigenstates are derived by applying the unitary operator $\hat{D}_{jk} \otimes \hat{1}$ to $|\Psi\rangle$. It is easy to check that $e^{(2\pi i/N)m(\hat{A}_1 - \hat{A}_2)}|\Psi_{jk}\rangle = \omega^{jm}|\Psi_{jk}\rangle$

and $e^{(2\pi i/N)n(\hat{B}_1+\hat{B}_2)}|\Psi_{jk}\rangle = \omega^{kn}|\Psi_{jk}\rangle$ with $|\Psi_{jk}\rangle = (\hat{D}_{jk} \otimes \hat{1})|\Psi\rangle$. The set $\{|\Psi_{jk}\rangle \mid j,k = 0,1,\ldots,N-1\}$ is a complete orthonormal basis of the Hilbert space $\mathcal{H}_N \otimes \mathcal{H}_N$. Therefore we have found that a completely entangled state of the Hilbert space $\mathcal{H}_N \otimes \mathcal{H}_N$ is the simultaneous eigenstate of the unitary operators $e^{(2\pi i/N)m(\hat{A}_1-\hat{A}_2)}$ and $e^{(2\pi i/N)n(\hat{B}_1+\hat{B}_2)}$ and conversely the simultaneous eigenstate of these operators is a completely entangled state.

Finally we consider a two-dimensional Hilbert case as a simple but important case. We set $\{|a_0\rangle, |a_1\rangle\} = \{|+\rangle, |-\rangle\}$, where $|\pm\rangle$ is the eigenstate of the Pauli matrix $\hat{\sigma}^x$ such that $\hat{\sigma}^x|\pm\rangle = \pm|\pm\rangle$. Then the discrete Fourier transformation yields $\{|b_0\rangle, |b_1\rangle\} = \{|0\rangle, |1\rangle\}$ with $\hat{\sigma}^z|0\rangle = |0\rangle$ and $\hat{\sigma}^z|1\rangle = -|1\rangle$. In this case, we obtain the Hermitian operators $\hat{A} = |-\rangle\langle-|$ and $\hat{B} = |1\rangle\langle 1|$. We also have the exponential unitary operators $e^{\pi i(\hat{A}_1-\hat{A}_2)} = \hat{\sigma}^x \otimes \hat{\sigma}^x$ and $e^{\pi i(\hat{B}_1+\hat{B}_2)} = \hat{\sigma}^z \otimes \hat{\sigma}^z$. Then the completely entangled states $|\Psi_{jk}\rangle$'s are the Bell states given by (2) and (3).

4 Concluding Remarks

In this paper, we have shown that a completely entangled state belonging to an $N \times N$ dimensional Hilbert space $\mathcal{H}_N \otimes \mathcal{H}_N$ is a simultaneous eigenstate of two unitary operators $e^{(2\pi i/N)m(\hat{A}_1-\hat{A}_2)}$ and $e^{(2\pi i/N)n(\hat{B}_1+\hat{B}_2)}$ with integers m and n, where the Hermitian operators $\hat{A}$ and $\hat{B}$ are complementary with each other. Conversely the simultaneous eigenstates of $e^{(2\pi i/N)m(\hat{A}_1-\hat{A}_2)}$ and $e^{(2\pi i/N)n(\hat{B}_1+\hat{B}_2)}$ become completely entangled states.

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